

Rigorously Testing Multialternative Decision Field Theory Against Random Utility Models

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Cognitive models of decision making aim to explain the process underlying observed choices. Here, we test a sequential sampling model of decision making, multialternative decision field theory (MDFT; Roe, Busemeyer, & Townsend, 2001), on empirical grounds and compare it against 2 established random utility models of choice: the probit and the logit model. Using a within-subject experimental design, participants in 2 studies repeatedly choose among sets of options (consumer products) described on several attributes. The results of Study 1 showed that all models predicted participants' choices equally well. In Study 2, in which the choice sets were explicitly designed to distinguish the models, MDFT had an advantage in predicting the observed choices. Study 2 further revealed the occurrence of multiple context effects within single participants, indicating an interdependent evaluation of choice options and correlations between different context effects. In sum, the results indicate that sequential sampling models can provide relevant insights into the cognitive process underlying preferential choices and thus can lead to better choice predictions.

Keywords: preferences, process models, MDFT, random utility models, context effects

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Recently, cognitive models of decision making aiming for a better explanation of human behavior by describing the processes underlying observed choices have received increasing attention. In contrast to this, many existing models of decision making do not account for cognitive processes but rather focus on just predicting observable outcomes. Surprisingly, so far, most comparisons between these outcome-oriented, static models against cognitive process models have been made on theoretical grounds, and rigorous tests of these models against each other have rarely been conducted on empirical grounds. One reason for this lack of empirical comparisons in the decision-making literature may be the difficulty of estimating the free parameters of many cognitive process models. Whether these models provide a feasible alternative thus remains somewhat unclear. Here, we propose a testable version of multialternative decision field theory (MDFT; Roe, Busemeyer, & Townsend, 2001), a prominent cognitive process model of choice, and compare it on empirical grounds with two established and widely used random utility models (RUMs) of choice that make no cognitive process assumptions, but merely

aim to predict decision outcomes. The main goal of the present work was to provide a rigorous empirical test of MDFT against the standard RUMs and illustrate people's interdependent evaluations of preferential choice options.

When trying to predict the outcome of a decision, one common approach relies on the theoretical framework of expected utility (von Neumann & Morgenstern, 1947). Provided that people's preferential choices obey certain choice axioms, this framework allows constructing a utility function such that their choices represent expected utility maximization. Due to this axiomatic approach, expected utility theories make deterministic predictions and cannot account for the probabilistic character of human choice (e.g., Mosteller & Nogee, 1951; Rieskamp, 2008). To account for randomness in people's choices, expected utility theory has been extended with an explicit error theory, leading to RUMs. Although RUMs do not aim for a description of the underlying cognitive processes that lead to the observable decision outcomes, they allow predicting the probability with which options are chosen (e.g., McFadden, 2001; Train, 2003). Standard RUMs assume that options are evaluated independently, such that the utility of any single option does not depend on other available options in the choice set.

Perhaps the two most prominent and widely used RUMs are the (multinomial) logit and probit models (e.g., Daganzo, 1980; Hausman & Wiese, 1978; Luce, 1959; McFadden, 1973; Thurstone, 1927). Both models have a long success record and are frequently applied in economics, psychology, consumer research, and related fields, including the domains of travel behavior (e.g., Adamowicz, Louviere, & Williams, 1994; Hensher, 1994; Train, 1978; Wardman, 1988), environmental behavior (e.g., Hanley, Wright, & Adamowicz, 1998; Roberts, Boyer, & Lusk, 2008), political choice behavior (e.g., Bowler, Karp, & Donovan, 2010; Karp, 2009;

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Nownes, 1992), consumer choices (e.g., Green & Srinivasan, 1978, 1990), or food choices (e.g., Gil & Sánchez, 1997; Loureiro & Umberger, 2005, 2007). For example, using probit models, Ryan and Farrar (2000) analyzed preferences in health care (e.g., treatment in a local clinic vs. treatment in a hospital), and Phillips, Maddala, and Johnson (2002) measured preferences for different HIV tests. Further, Loureiro and Umberger (2007) analyzed the importance U.S. consumers assign to the country-of-origin labeling and traceability of beef, and Koistinen et al. (2013) investigated the impact of fat content and carbon footprint information on the relative preferences of Finns for minced meat—both using logit models. Presumably, the widespread use of these models is largely due to their ease of implementation and estimation (Train, 2003).

In contrast to these outcome-oriented models, many cognitive approaches to decision making aim for a description of the processes that underlie observable choices (e.g., Busemeyer & Diederich, 2002; Lewandowsky & Farrell, 2011). Within this category, sequential sampling models represent a particularly promising approach (e.g., Busemeyer & Townsend, 1993; Scheibehenne, Rieskamp, & Gonzalez-Vallejo, 2009; Usher & McClelland, 2004). These models have a long tradition in psychology, explaining, for instance, memory and perception processes (e.g., Ratcliff, 1978; Townsend & Ashby, 1983; Vickers, 1970). Sequential sampling models of preferential choice often assume that people accumulate information or evidence about the available options over time and that a choice is made once the accumulated evidence passes a decision threshold. One sequential sampling model that has been suggested as a powerful theory for modeling preferential choices is MDFT (Roe et al., 2001). MDFT aims to explain how preferences are formed and how they evolve over time. The model assumes that at any point in time during the deliberation process, a preference is formed for each available option until the accumulated evidence of one option reaches a predefined decision threshold. Each temporary preference state represents the integration of all previous states. The preferences are formed on the basis of an attention-switching process that assumes attention between the attributes of the options (e.g., the price or the quality of consumer products) switches from one point in time to the next in an all-or-nothing manner. The integration of all previous preference states is subject to a decay function that accounts for imperfect preference recollection such that parts of the previous preference states are lost during the integration (Johnson & Busemeyer, 2010). MDFT further assumes that at each point in time, options are evaluated relative to each other by comparing the attribute values. Finally, the theory assumes that options inhibit each other as an increasing function of their similarity, related to the concept of lateral inhibition (McClelland & Rumelhart, 1981). That is, closer (i.e., more similar) options inhibit each other more strongly than more distant options.

Besides having the ability to advance the theoretical understanding of cognitive processes, MDFT also promises a higher predictive accuracy, because it takes situational aspects into account such as time pressure, cognitive load, or similarities between options that RUMs ignore (e.g., Diederich, 2010; Pettibone, 2012; Roe et al., 2001). Such a cognitive and process-driven approach has often been called for in the choice literature (e.g., Chandukala, Kim, Otter, Rossi, & Allenby, 2007; Otter et al., 2008; Reutskaja, Nagel, Camerer, & Rangel, 2011). Although promising, complex cogni-

tive models with several free parameters such as MDFT are inherently more flexible in fitting any observed data compared with more parsimonious RUMs such as the logit and probit models. Therefore, the question arises whether sequential sampling models such as MDFT still yield an increase in predictive accuracy when model complexity is taken into account.

Past research indicated that MDFT can explain a number of systematic violations of standard RUMs based on theoretical grounds (e.g., Busemeyer, Barkan, Mehta, & Chaturvedi, 2007; Rieskamp, Busemeyer, & Mellers, 2006). However, rigorous comparisons of these models on empirical grounds are lacking. Thus, it is an open empirical question as to what extent the increased model complexity of MDFT actually yields improved predictive accuracy as compared with the more parsimonious RUMs. An empirical test will also clarify whether the frequent application of RUMs in many domains is justified. Such a test requires that MDFT's free parameters can be estimated from empirical data. Estimating a model using empirical data differs qualitatively from illustrating specific predictions from a set of given parameter values. Surprisingly, to our knowledge, despite MDFT's prominence, empirical studies designed to estimate the model's parameters in empirical studies are lacking. Presumably, this is because in its original form, the choice probabilities that MDFT predicts are not analytically specified but need to be derived from time-consuming process simulations. Similarly, Otter et al. (2008, p. 259) pointed out that due to "specification issues and computational challenges," the estimation process of MDFT on the basis of empirical data is difficult (see also Soltani, De Martino, & Camerer, 2012).

In the following, we meet this challenge by providing a version of MDFT that can be applied to observed choice data. On the basis of this, we describe an empirical study in which we tested and compared MDFT against two of the presumably most prevalent RUMs in the choice literature: the logit and the probit model. We tested the models in choice situations in which participants repeatedly chose consumer products out of a set of three options (Study 1). In Study 2, we tested and compared the models in choice situations that entailed so-called context effects (described in detail below), which presumably allow MDFT to perform in its highest gear. We provide a detailed description of MDFT and how to estimate its parameters on the basis of empirical data. In the next section, we provide a specification of the logit and probit models.

The Logit and Probit Models

The logit and probit models both assume that options are compared on the basis of their respective subjective utilities and that the option with the highest utility is most likely chosen. The two models differ in their error theories, which lead to differences in the options' utilities. For a single decision maker, the utility of an option i out of a set of J options is defined as:

$$U_i = V_i + \varepsilon_i, \quad (1)$$

where V_i indicates the subjective value of that option, and the error term ε_i represents a random variable with $\varepsilon = [\varepsilon_1, \dots, \varepsilon_i, \dots, \varepsilon_J]$. The logit model assumes that the error ε is extreme value distributed, whereas the probit model assumes normally distributed errors (Train, 2003). For both models, the subjective value V_i is the product of a vector β that contains the weights (i.e., the impor-

tance) given to the m attributes of an option, and the value vector X_i with $X = [X_1, \dots, X_i, \dots, X_J]$ containing the attribute values of option i :

$$V_i = \beta X_i. \tag{2}$$

The probability p of choosing option i is defined as:

$$p_i = \text{probability}(V_i + \varepsilon_i > V_j + \varepsilon_j; \forall j \neq i) \\ = \int_{\varepsilon} I(V_i + \varepsilon_i > V_j + \varepsilon_j; \forall j \neq i) f(\varepsilon) d\varepsilon, \tag{3}$$

where $I(\cdot)$ is an indicator function that takes the value 1 if the condition is fulfilled; otherwise it has a value of 0 (see Train, 2003 for details).

For the logit model, but not the probit model, a closed form representation of this integral exists:

$$p_i = \frac{e^{V_i}}{\sum_{j=1}^J e^{V_j}}. \tag{4}$$

Thus, the logit model has m free parameters, one for each attribute weight. To make the probit model mathematically identifiable, different parameterizations exist (Train, 2003). Here, we chose to fix one of the weight parameters, which leaves the probit model with $m - 1$ attribute weight parameters and one free parameter ν that specifies the variance of the normal distributed error term ε .

MDFT

In MDFT, the preference for each option at any point in time t is captured by a preference vector P_t , referred to as a preference state containing the preferences of all J options. P_t integrates all previous preference states and adds the current evaluation or valence V_t of the options according to the following updating process:

$$P_t = SP_{t-1} + V_t. \tag{5}$$

The process described in Equation 5 continues until one option reaches a predefined decision threshold (a so-called internal stopping rule) or when the decision time is up (external stopping rule). Here, S is a feedback matrix that reflects to what extent the previous preference states for the given options are memorized (diagonal elements) and how the options influence each other, depending on their distances in the attribute space (off-diagonal elements). It is defined as:

$$S = \delta - \varphi_2 \times \exp(-\varphi_1 \times D^2), \tag{6}$$

where δ is an identity matrix, the decay parameter φ_2 determines the diagonal elements of S , and the sensitivity parameter φ_1 determines the similarity as a function of the distance D between the options in the attribute space (cf. Hotaling, Busemeyer, & Li, 2010). MDFT assumes that people evaluate each option relative to each other option; that is, people compare an option's attribute value with the corresponding value of the other option on that attribute. This process leads to interdependent evaluations of choice options and is reflected in the valence vector V_t . V_t can be decomposed into three matrices and an error component:

$$V_t = CMW_t + \varepsilon, \tag{7}$$

where C is a contrast matrix to compute the advantages or disadvantages of each option relative to the alternative options (see Busemeyer & Diederich, 2002, for the general formula of C). The value matrix M contains the attribute values of each option (comparable to the value matrix X in the logit and probit models), and the weight vector W_t represents the attention or importance weights for each attribute. Over time, each attribute informs the valence formation proportional to its importance (comparable to the weight vector β in the logit and probit models). If the updating process is omitted, the probit model becomes a special case of MDFT; that is, it is nested within MDFT (see the supplemental materials for the mathematical details). For the full description of how to determine MDFT's choice probabilities, see also Appendix B of Roe et al. (2001).

Estimating MDFT

To estimate the free parameters of MDFT based on observed choice data, one must solve Equation 5. Unless certain auxiliary assumptions are imposed, this requires laborious numerical integration techniques, as analytic solutions are not readily available (see also Trueblood, Brown, & Heathcode, 2013; Tsetsos, Usher, & Chater, 2010). Furthermore, as the predictions of MDFT depend on the similarities between the options in the attribute space, a distance function must be specified (Hotaling et al., 2010; Tsetsos et al., 2010). To address these requirements, we made the simplifying assumption that decision makers only decide once the preference state for the different options stabilizes, that is, converges to a specific value. Therefore, we set $t \rightarrow \infty$.¹ Although this approach sacrifices MDFT's ability to make predictions about decision time, no decision threshold or maximum decision time needs to be specified, and an analytical solution to calculate the choice probabilities exists (see Appendix A for the mathematical derivations).²

As mentioned above, MDFT assumes that preferences partly depend on the similarity of the available options. To determine the effect of these similarities, a function is needed that quantifies these influences. Existing distance functions (e.g., Hotaling et al., 2010) assume equal weighting of the attributes or consider a maximum of only two attributes. To allow for more than two attributes and flexible attribute weights, we included a generalized distance function that describes the distance between any two options in the multidimensional space with indifference vector(s) and a dominance vector (Berkowitsch, Scheibehenne, Matthäus, & Rieskamp, 2013). The indifference vectors specify how much importance a person gives to the different attributes, and the dominance vector specifies the preferential relationship between the options, that is, whether an option dominates another option or not (Hotaling et al., 2010; Tversky, Sattah, & Slovic, 1988; Wedell, 1991). The underlying idea here is that the psychological distance decreases more slowly when moving from one option to another along the line of preferential indifference (i.e., the indifference direction), than along the line of preferential dominance (i.e., the dominance direction). To account for different weighting of the dominance and indifference directions, only the dominance

¹ Technically, this requires the eigenvalues of the feedback matrix to be smaller than 1 (Busemeyer & Diederich, 2002).

² For the R-code to estimate the parameters of MDFT, see <http://goo.gl/Uu6eYW>.

vector is multiplied by a weight $wd > 1$, as suggested by Huber, Payne, and Puto (1982) and also by Hotaling et al. (2010; for more details on the generalized distance function, see Appendix B and Berkowitsch et al., 2013).

Taken together, this specification of MDFT requires the estimation of $m - 1$ attention weight parameters for each product attribute, the variance parameter ν of the normally distributed error component, the sensitivity parameter φ_1 at which similarity declines with distance between the options, and the decay parameter φ_2 specifying how quickly the preference state decays during the updating process. In Study 1, the dominance parameter wd was not estimated because in this study, dominated options were eliminated from the choice sets. In contrast, in Study 2 in which dominated options were part of the choice sets, the parameter was estimated (see Appendix C for the constraints on the parameters). Implemented this way, for example, a choice between three options with five attributes requires the estimation of seven (eight with wd) parameters, as compared with five free parameters for both the logit and the probit model.

Study 1: Comparing MDFT With RUMs

The aim of Study 1 was to test whether the model parameters of MDFT can be estimated from observed choice data when certain simplifying assumptions are met (see above). This requirement is necessary for comparing the predictive accuracy of MDFT with that of alternative choice models such as RUMs on empirical grounds, which was another goal of Study 1. Toward these goals, we conducted two consecutive experiments in which participants repeatedly chose their favorite digital camera out of a set of three available options. In the first calibration experiment, we compared how well the models could describe participants' choice behavior. In the second generalization experiment, we used the results from the calibration experiment to create new choice sets for which MDFT and the RUMs made maximally different predictions. This generalization test allows for a rigorous comparison between the two models that takes model complexity into account without the need to reestimate the models' parameters (Busemeyer & Wang, 2000).

Method

Participants. Thirty university students (66% female, mean age = 24 years) participated in each of the two experiments. The average participant took about 13 and 15 min, respectively, to complete the experiments and received a show-up fee of 3 Swiss francs (about \$3 U.S.; calibration experiment) and 10 Swiss francs (about \$11 U.S.; generalization experiment).

Procedure and design. To incentivize participants' choices, all participants were entered into a lottery where they had a chance to win one of the cameras they chose in the experiment (or a very similar one). Each camera was described by five attributes: megapixels (4, 6, or 8), optical zoom (3 \times , 5 \times , or 10 \times), picture quality (good vs. very good), screen size (2 vs. 3 in.), and availability of optical image stabilizer (yes vs. no). This led to a total of 72 distinct cameras that were presented in 72 different randomized sets of three cameras each. To create the choice sets for both the calibration and the generalization experiment, we first created all possible 357,840 (i.e., $72 \times 71 \times 70$) sets of three cameras. Next,

we deleted all sets with dominant options. From the remaining pool, we randomly selected 72 choice triplets.

On the basis of participants' choices in the calibration experiment, we estimated the models' parameters for each individual participant. On the basis of these parameters, we selected 72 new choice triplets for which the models made maximally different predictions. To select this generalization set, we generated predictions from each model for 1,000 randomly sampled choice triplets. Model predictions were generated using bootstrap methods in which we randomly sampled (with replacement) sets of parameters from the participants in the calibration experiment (see Busemeyer & Wang, 2000). Triplets for the generalization experiment were selected by finding those 72 triplets for which the average city block distance between the models' predictions was highest both between MDFT and the logit model and between MDFT and the probit model. The city block distance sums the absolute difference of the predicted mean probability between the models for each option, thus providing a single distance value for each choice triplet (Attneave, 1950).

Model comparison. As a first comparison step in the calibration experiment, we estimated the free parameters of all models using maximum likelihood methods. The search space for the parameter values was restricted within a reasonable range (see Appendix C for details of the range for each parameter). To take the models' complexities into account, we determined the Bayesian information criterion (BIC) for each model (Raftery, 1995). We used the difference in BIC values to determine the relative posterior probability that a model generated the data for each individual participant. These probabilities were scaled so that they added up to 1 across models (Raftery, 1995). Because MDFT has more free parameters, it gets penalized more strongly than the logit and the probit model. Additionally, we tested the models against a baseline model assuming a random choice between the three options with a probability of 1/3. Naturally, any reasonable model is expected to out-compete the baseline model in predicting the observed choices. In the next step, we applied a generalization test to compare the accuracies of the models' predictions for the generalization experiment that were based on the estimated models' parameters from the calibration experiment (Busemeyer & Wang, 2000).

Results

Descriptive results. To investigate the agreement between participants' choices within each of the two experiments, we calculated the relative frequency of the most popular option within each of the 72 choice triplets. On average, 66% and 58% of the participants chose the same digital camera in each triplet in the calibration and generalization experiment, respectively.

Model comparison calibration experiment. The mean log-likelihood μ_{LL} (with $\mu_{LL} = 0$ indicating perfect fit) across all participants was highest for MDFT ($\mu_{LL} = -48.65$), followed by the logit model ($\mu_{LL} = -50.00$) and the probit model ($\mu_{LL} = -50.57$), whereas the μ_{LL} of the baseline model was -79.10 , illustrating that all three models made more accurate predictions than the baseline model. Comparisons of the log likelihoods on an individual level indicated that 22 (73%) participants were best described by MDFT. The remaining eight (27%) partic-

ipants were best described by the logit model, and nobody was best described by the probit model.

Figure 1 shows the results of the model comparison based on the BIC. Here, relative model probabilities were classified as weak (.33–.60), positive (.60–.91), strong (.91–.99), and very strong (>.99; adapted from Raftery, 1995, for three models). The figure illustrates that when taking the models’ complexities into account, the advantage of MDFT over the RUMs diminishes: Now, 83% (25) of participants were best described by the logit model, followed by 13% best described by the probit model and 3% by MDFT.

As an additional illustrative measure of absolute model fit, we examined the percentage of choices in which the option with the highest predicted probability was chosen by the participants. The percentages of correctly predicted choices were comparable across models (MDFT: 74%, logit: 73%, probit: 72%) and considerably higher than the baseline model (33%).

The correlations between the estimated attribute weights across models indicate that all three models yield comparable attribute weights, $r_{\text{logit,probit}} = .82$ ($SD = .38$), $r_{\text{MDFT,probit}} = .79$ ($SD = .38$), $r_{\text{MDFT,logit}} = .94$ ($SD = .17$).³ This result indicates that all three models were able to identify the importance the participants gave to the different attributes (see the supplemental materials for the estimated ranges of the parameters). For instance, on average, the attribute of picture quality (w_3) was identified as having the largest importance for digital cameras by all three models (see Tables S1–S3 in the supplemental materials).

Model comparison generalization experiment. As an initial measure of out-of-sample accuracy, we analyzed how well each of the models predicted choices in the generalization experiment. For all but five choices, the three models agreed in their prediction of what option would most likely be chosen. Therefore, the mere percentage of how often the most frequently chosen option was predicted correctly was quite similar (MDFT: 82%, probit: 81%, logit: 75%). Nevertheless and decisively, the probabilities by which the models predicted the choices differed substantially. To take these differences into account, we compared the models on the basis of the mean log likelihood of their probabilistic predictions. Here, MDFT ($\mu_{LL} = -6.81$) and the probit model ($\mu_{LL} = -6.50$) were most accurate, whereas the baseline ($\mu_{LL} = -9.79$) and the logit model were least accurate ($\mu_{LL} = -11.99$). Analyses on the basis of single choices indicated that MDFT and the probit model

predicted people’s choices most accurately in 31 (43%) and 32 (44%) of the 72 choice triplets, respectively. A closer look at the predictions for each triplet revealed that, on average, the choice probabilities predicted by the logit model were most extreme—resulting in either very accurate or very inaccurate predictions. For the five triplets where the three models made qualitatively different predictions, MDFT predicted participants’ choices most accurately, as indicated by the mean log likelihood for these five options ($\mu_{LL_MDFT} = -10.61$, $\mu_{LL_probit} = -14.29$, $\mu_{LL_logit} = -16.44$, and $\mu_{LL_baseline} = -16.09$).

Discussion

In Study 1, we outlined how MDFT could be estimated on the basis of empirical data, and we tested it against two competing models in two consecutive choice experiments. In the first calibration experiment, MDFT provided a better fit to the observed choice data for most participants. However, when taking model complexity into account, the advantage of MDFT over the more parsimonious RUMs largely disappeared, suggesting that the advantage of MDFT was mainly due to its higher flexibility in fitting the data. The crucial second generalization experiment revealed that MDFT outperformed the logit model and rivaled the probit model, surpassing the latter for cases in which MDFT made qualitatively different predictions. Thus, when predicting preferential choices, MDFT was not necessarily better, but also not worse than the RUMs in the context at hand.

As outlined in the introduction, one advantage of MDFT over RUMs is that it can take systematic influences of the context and similarities between the available options into account. Previous research has identified a number of situations in which option evaluations systematically depended on the context of other available options (Huber et al., 1982; Huber & Puto, 1983; Simonson & Tversky, 1992; Slovic & Tversky, 1974). These findings violate the assumption that options are evaluated independently, sometimes referred to as the *independence of irrelevant alternatives* (IIA) principle (cf. Rieskamp et al., 2006). According to this principle, the ratio of the choice shares of any two options stays constant when another option is added or removed from the set of options (Luce, 1959). Systematic violations of the IIA principle can be elicited by selectively adding options to an existing set of options (e.g., Huber et al., 1982; Huber & Puto, 1983; Tversky 1972a, 1972b). In contrast to RUMs, MDFT provides a cognitive explanation of when and how systematic violations of the IIA principle occur, and in theory, it can predict these violations. Thus, in a choice situation in which systematic context effects are likely to occur, MDFT should outperform RUMs.

Because of the way we created the choice sets in Study 1, we did not expect any systematic violations of the IIA principle. Thus, even though the experiment may resemble common choice situations in real life and in the lab, presumably it did not allow MDFT to perform in its highest gear. Therefore, in the following study, we tested MDFT against RUMs for situations in which violations of the IIA principle were expected to occur frequently.

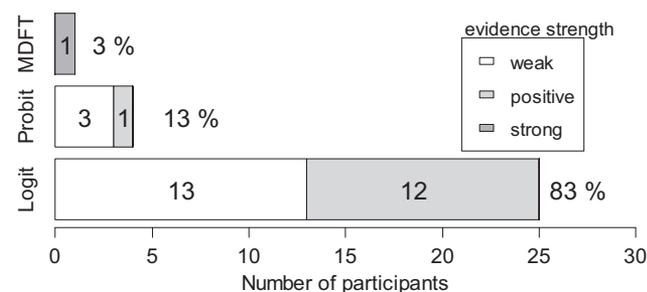


Figure 1. Model comparison of the relative model probabilities based on their Bayesian information criteria. Numbers in the bars indicate how many individuals fell into a specific level of evidence strength. The percentages indicate what proportions of participants were assigned to each model. MDFT = multialternative decision field theory.

³ Prior to calculating the correlations, the attribute weights of the logit model were standardized with respect to their standard deviations (Menard, 2004).

Study 2: Comparing MDFT With RUMs Using Context Effects

Three well-known context effects that systematically violate the IIA principle are the so-called attraction, compromise, and similarity effects (described in more detail below). There is a substantial body of research on these effects (for a review, see Heath & Chatterjee, 1995), yet so far, almost all empirical studies have focused on each of the single effects in isolation or used a between-subjects experimental design to elicit the effects; however, there are no theoretical reasons for this separation. If these context effects are due to interdependent evaluations of choice options, multiple context effects should also occur for the same person. Indeed, using an inference and a perceptual task, Trueblood and colleagues reported empirical findings showing that all three context effects can occur within the same experimental design (Trueblood, 2012; Trueblood, Brown, Heathcote, & Busemeyer, 2013) and within a single person (Trueblood, Brown, & Heathcote, 2013). Similarly, Tsetsos, Chater, and Usher (2012) elicited the attraction and the similarity effect within individuals using a risky choice task. Therefore, it seems plausible that comparable effects could also occur for preferential choices, which is the domain in which context effects presumably have received most attention in the past. To find out, in Study 2, we aimed to test whether all three context effects can occur simultaneously for the same person in a preferential choice task. In the following, we describe the three context effects in more detail:

The *attraction effect* refers to a choice situation in which adding an option dominated by one of the existing options increases the choice share of the dominating option (Huber et al., 1982; Huber & Puto, 1983). This effect additionally represents a violation of the so-called regularity principle, according to which the absolute choice share of an option can only stay constant or decrease when a new option is added to a set of options. Another well-documented context effect is the *compromise effect*, which can occur when a third option is added to an existing set such that one of the original options appears as a compromise, thereby increasing its relative choice share (Simonson & Tversky, 1992; Tversky & Simonson, 1993). Finally, the *similarity effect* is based on the observation that adding a choice option that is similar to one but not to the other option has been shown to increase the relative choice share of the dissimilar option, presumably because the similar options “compete” more strongly with each other (Tversky, 1972a, 1972b).

Model Comparison

In theory, MDFT can simultaneously account for all three of these context effects by incorporating different cognitive mechanisms given a specific set of parameter values (Roe et al., 2001). However, it is unclear whether this specific set of parameters is also suitable to accurately predict people’s preferential choices. In Study 2, we sought to test how well MDFT can predict empirical choice data when people’s preferential choices are affected by different context effects. Like in Study 1, the main question in this situation was again whether MDFT provides a better explanation of the data as compared with RUMs that cannot account for systematic context effects. Thus, we predicted that the advantage of MDFT over RUMs would increase as decision makers became more prone to context effects.

To test the advantage of MDFT over RUMs in cases in which multiple context effects occur within a single individual, we created choice sets that increased the chances of observing attraction, compromise, and similarity effects within the same person. As outlined in detail below, we did this by systematically varying the position of choice options in the attribute space. We compared the models against each other on the basis of BIC and cross-validation (Browne, 2000; Stone, 1974).

Method

Participants. Forty-eight students (67% female, mean age = 24 years) from the University of Basel, Switzerland, participated in Study 2 in exchange for 25 Swiss francs (CHF). The study took 57 min on average.

Procedure and design. Participants repeatedly chose a consumer product described on two attributes. The study consisted of two consecutive sessions. In the first session, we aimed to find pairs of options to which individuals were indifferent. In the second session, we systematically added new choice options to that initial pair to elicit different context effects within each participant.

To identify indifference pairs in the first session, participants repeatedly filled in missing attribute values (e.g., price) so that two products (e.g., a heavier and a lighter racing bike) became equally attractive (Carmon & Simonson, 1998). Prior to this matching task, we provided participants with a short explanation of the attributes and the possible value range for each of the six products (see Figure 2 for an example of this task and Table 1 for a list of the utilized products and their attributes). With this matching procedure, we created 108 pairs of options (i.e., 18 per product pair) to which each single participant was expected to be indifferent. We refer to these pairs as the *matched* options.

The second session took place a few days later and involved a choice task that was similar to Study 1. For each participant, the previously matched Options A and B were combined with one new option that was carefully placed within the attribute space (see

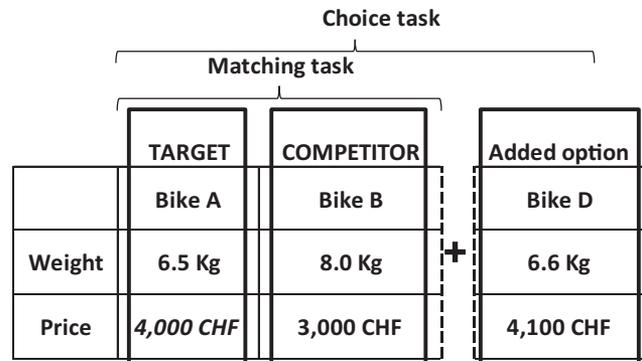


Figure 2. Example of a matching and a choice task for racing bikes. The price of Bike A was blank before participants matched the italic number (e.g., 4,000). Here, the choice task is intended to elicit an attraction effect (see also choice triplet BDA in Figure 3a).

Table 1
Products and Attributes Presented in Study 2

Product	Attribute 1	Attribute 2
Color printer	Printing speed in pages per minute	Price in Swiss francs
Digital camera	Picture quality in megapixels	Memory space in gigabytes
Notebook computer	Weight in kilograms	Battery longevity in hours
Racing bike	Weight in kilograms	Price in Swiss francs
Vacuum cleaner	Suction power in watts	Price in Swiss francs
Washing machine	Water consumption in liters	Life cycle in years

Figure 3).⁴ We balanced the positions of the new options, such that participants were faced with six attraction, compromise, and similarity triplets for each matched pair. We refer to the matched option expected to be chosen more frequently on theoretical grounds as the *target*. The remaining matched option is referred to as the target’s *competitor*. We balanced the choice triplets for each participant such that for half of the choice triplets, the target was Option A (see Figure 3a), and for the other half it was Option B (see Figure 3b).

For attraction triplets (BDA and DBA in Figure 3a and 3b, respectively), the dominated option was placed at a distance of about 10% from the target, orthogonal to the indifference line AB. For the compromise triplets (BAC and CBA in Figure 3a and 3b, respectively), the extreme option was placed along the indifference line such that the target had the same distance to the competitor and to the extreme option. We chose a distance of about 10% of AB in the indifference direction between the competitor and the new similar option to create similarity triplets (SBA and BAS in Figure 3a and b, respectively). Due to rounding, half of the intended similarity choice triplets became attraction choice triplets (i.e., the added option did not lie on the indifference line) and were therefore excluded from the analysis, which reduced the statistical power to analyze the similarity effect. However, the design was still balanced in the sense that for half of the triplets, Option A was the target, and for the other half, Option B was the target.

Using this balanced design, we compared the relative choice shares of the three options and subsequently tested for each participant, whether adding a new option to the previously matched options influenced the *relative choice share of the target (RST)*, defined as

$$RST = \frac{N \text{ targets}}{N \text{ targets} + N \text{ competitors}}, \tag{8}$$

where *N* targets indicates how often a participant chose the target option and *N* competitors indicates how often the competitor was chosen. The measure was calculated separately for the attraction, compromise, and similarity triplets. Due to the way the choice sets were created, the target and the competitor will be chosen about equally often if no context effect occurs (i.e., *RST* = .50). An *RST* value larger than .50 indicates an increase of the target choice share relative to that of the competitor, and hence a systematic context effect. The order of the products as well as the order of the choice triplets within products was randomized.

Results

Occurrence of context effects. Did adding the third option to the previously matched pairs change people’s preferences? Figure

4 shows a histogram of the choice shares of the target, the competitor, the added option, and the *RST* averaged across all participants and products, separately for the attraction, the compromise, and the similarity triplets. As can be seen from the figure, on a descriptive level, the *RST* of all three types of triplets exceeded .50.

Next, we tested whether the mean *RST* across all participants was larger than .50 for any of the three context situations. As a statistical measure, we calculated the 95% highest posterior density interval (HDI) representing the most credible *RST* values using Bayesian statistics (Kruschke, 2011a, 2011b). If the HDI₉₅ excludes .50, one can infer a reliable context effect. We adapted a hierarchical Bayesian model following Kruschke (2011b), assuming uniform prior probability distributions across the parameter range.

Results of that analysis indicated a strong and reliable effect for the attraction triplets with an HDI₉₅ of .60–.66 (mean = .63). Essentially, 100% of the posterior probability density was above the critical threshold of .50. Likewise, for the compromise triplets, a reliable effect was observed with an HDI₉₅ of .52–.63 (mean = .58), with 99% of the density being above .50. For the similarity triplets, the HDI₉₅ of .48–.59 (mean = .54) included the critical value of .50, although 93% of the posterior density was above .50, suggesting a weak similarity effect.

Are participants who are prone to one context effect also prone to another? In total, nine participants (19%) showed an *RST* value higher than .50 for all three context effects. On an individual level, the individual HDI₉₅ for each of these participants did not exclude .50 for all three context effects, which is probably due to having too little statistical power on the individual level. For example, if these nine participants are analyzed post hoc as a group of subjects, the HDI₉₅ would exclude .50 for all three effects. Figure 5 plots the *RST* across individual participants. The figure shows that the *RST* was positively correlated for the attraction and compromise triplets (*r* = .49, *SD* = .10), indicating that participants who showed the attraction effect also often showed the compromise effect. Interestingly, there was a strong negative correlation for the *RST* between the similarity and the attraction triplets (*r* = -.53, *SD* = .10) and between the similarity and the compromise triplets (*r* = -.58, *SD* = .15). This indicates that people who showed either the attraction or the compromise effect rarely showed the similarity effect.

Model comparison. To compare the predictive accuracies of MDFT against RUMs, we estimated the free parameters of the models using maximum likelihood techniques similar to Study 1.

⁴ Only correctly matched product pairs (i.e., the missing price of a lighter bike must be filled in as more expensive than the price of a heavier bike) were presented in the subsequent choice task.

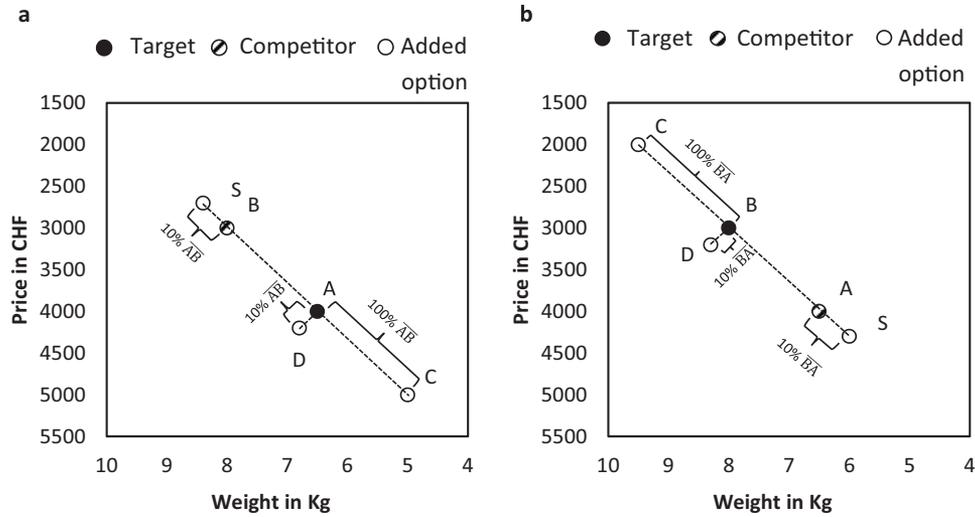


Figure 3. Illustrative example of the choice task, where either Option A (a) or Option B (b) is the target, depending on the position of the added option. For each participant, two matched options A and B were presented with one of the individually calculated decoys C, D, or S.

Results indicated that the mean log likelihood across participants was highest for MDFT ($\mu_{LL} = -57.81$), followed by the logit model ($\mu_{LL} = -66.49$) and the probit model ($\mu_{LL} = -71.08$). All three models predicted the observed choices better than a baseline model predicting random choices ($\mu_{LL} = -91.21$). Comparing the log likelihoods within each individual revealed that MDFT outperformed both the logit and the probit model for each of the 48 participants.

Additional analyses based on the mean log likelihood across participants and triplets revealed the highest difference between MDFT and the RUMs occurred for attraction triplets (MDFT: $\mu_{LL} = -0.61$, logit: $\mu_{LL} = -0.73$, probit: $\mu_{LL} = -0.80$), fol-

lowed by compromise triplets (MDFT: $\mu_{LL} = -0.74$, logit: $\mu_{LL} = -0.86$, probit: $\mu_{LL} = -0.90$), and a weaker difference for similarity triplets (MDFT: $\mu_{LL} = -0.81$, logit: $\mu_{LL} = -0.84$, probit: $\mu_{LL} = -0.89$).

To test whether MDFT still outperforms the RUMs if the models' complexities are taken into account, we calculated the relative model probabilities on the basis of the BIC, similar to Study 1. This analysis revealed that the choice behavior of 31 participants (65%) was best described by MDFT, and 12 participants (25%) were best described by the logit model. The remaining five participants (10%) were best described by the probit model. For a substantial number of participants, the obtained relative

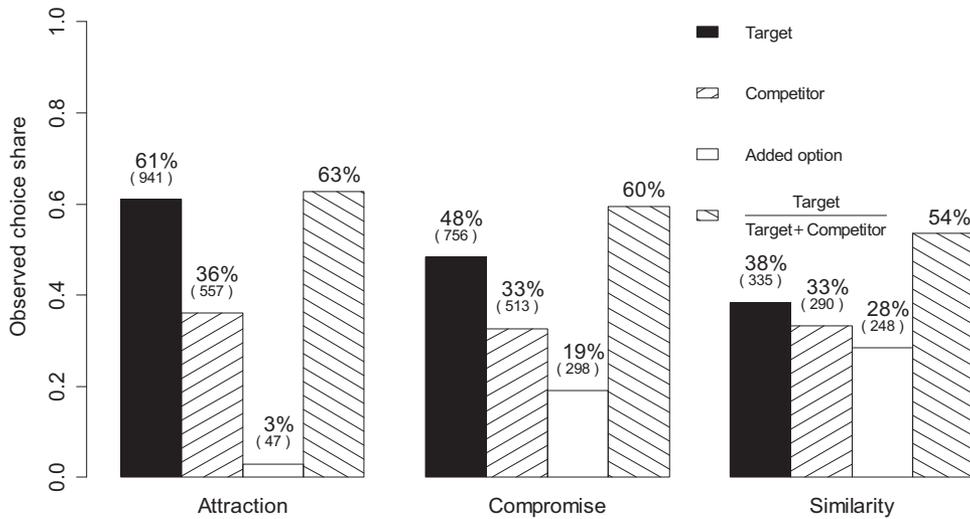


Figure 4. Observed choice shares across all participants' choices for the target, the competitor, the added option, and the relative choice shares of the target for the attraction, compromise, and similarity choice triplets. The percentages of the first three columns within each category add up to 100% except for the similarity choice triplets (due to rounding). Absolute numbers are shown in parentheses.

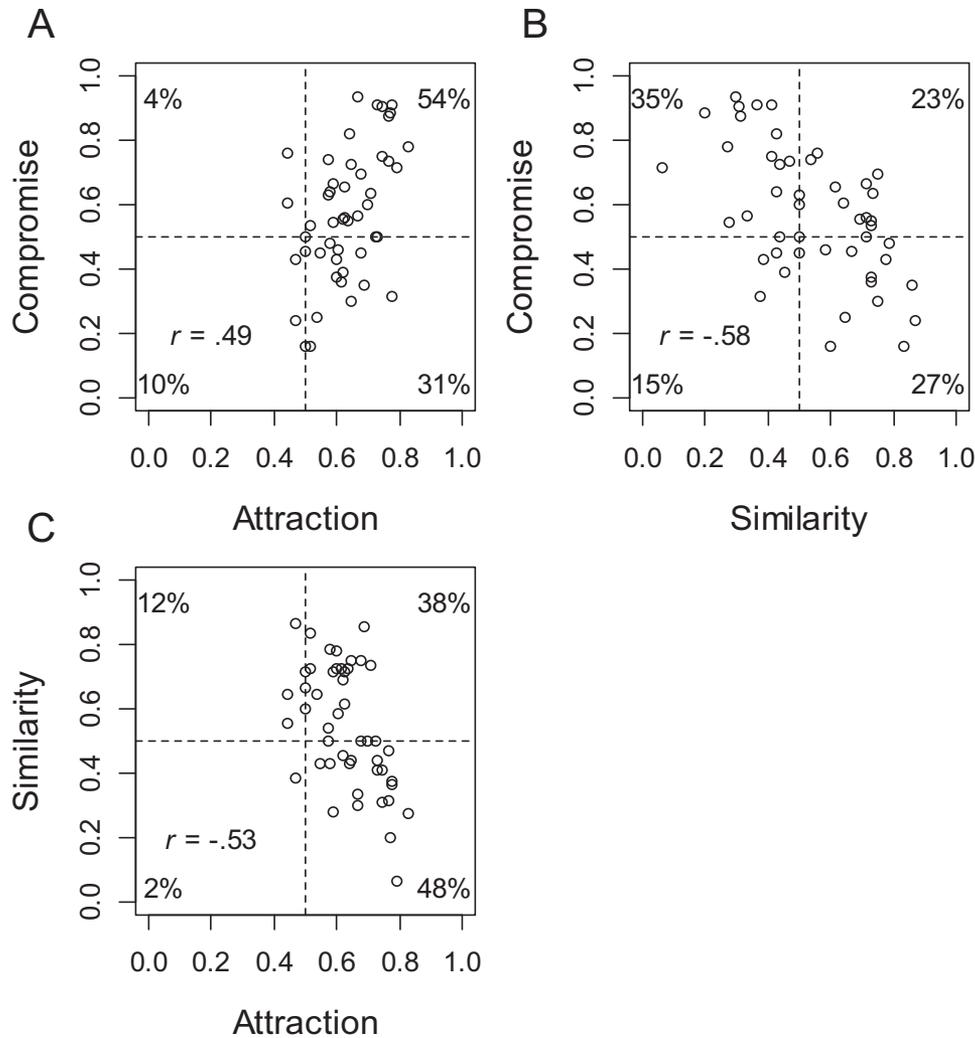


Figure 5. Plot of the relative choice shares of the target for each participant. Within each panel, dots in the upper-right quadrant indicate participants who are prone to both context effects, and dots in the upper left and lower right indicate participants who are prone to only one context effect. Dots in the lower left indicate participants who are not prone to either of the two context effects.

model probabilities indicated very strong evidence for MDFT. In contrast, for those participants who were best described by the logit model, the evidence was mostly weak. Figure 6 summarizes these results.

To test whether the 13 participants for whom the relative model probabilities indicated very strong evidence for MDFT were more prone to context effects than the remaining 35 participants, we contrasted the mean *RSTs* of the two groups. Results indicated that these 13 participants had higher *RSTs* for the compromise triplets (68% vs. 53%, $HDI_{.95} .02-.26$, mean = .14, 99% of the $HDI > 0$), but not for the attraction triplets (64% vs. 63%, $HDI_{.95} -.05$ to .08, mean = .02, 70% of the $HDI > 0$) or the similarity triplets (54% vs. 54%, $HDI_{.95} -.12$ to .11, mean = .01, 46% of the $HDI > 0$).

As an alternative model selection criterion, we also compared the models using cross-validation (Browne, 2000; Stone, 1974). As a first step of this analysis, we split the choice data of each

participant into two parts: a calibration sample to fit the model parameters and a validation sample to test the model predictions. To create the validation sample, we randomly drew two attraction, two compromise, and two similarity triplets. The remaining data were used for the calibration sample. To ensure that the results were not influenced by this random selection, we repeated this procedure four times. Next, we compared the models' log likelihoods for both the validation and the calibration set, averaged across the four samples. In all four samples, participants' choices in the calibration set were best described by MDFT (μ_{LL} across samples = -52.86), followed by the probit model (μ_{LL} across samples = -60.49) and the logit model (μ_{LL} across samples = -61.27). More importantly, the results showed that MDFT also made the most accurate predictions for the validation sample (μ_{LL} across samples = -5.23), followed by the logit model (μ_{LL} across samples = -5.82), the probit model (μ_{LL} across sam-

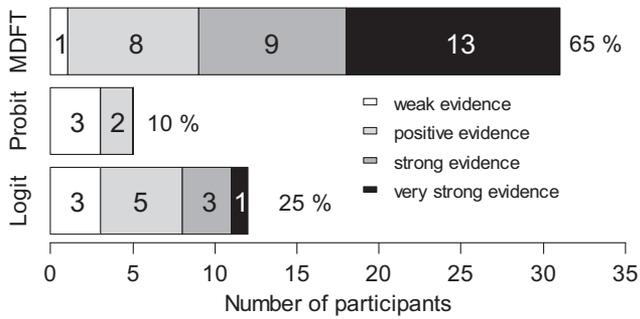


Figure 6. Model comparison of the relative model probabilities based on their Bayesian information criteria. Numbers in the bars indicate how many individuals fell into a specific level of evidence strength. The percentages indicate what proportions of participants were assigned to each model. MDFT = multialternative decision field theory.

ples = -5.90), and the baseline model (μ_{LL} across samples = -6.59). These results were also reproduced on the individual level, where most participants were most accurately predicted by MDFT (mean across samples = 68%), followed by the probit (mean across samples = 20%) and logit (mean across samples = 12%) models.

Besides allowing these quantitative model comparisons, the data on hand also provide the opportunity to test to what extent the models predict the observed correlations between the context effects. As a first step toward this analysis, we calculated the models' predicted probabilities for each participant of choosing the target and competitor, across all attraction, compromise, and similarity triplets. Next, we derived the *predicted relative choice share of the target (PRST)* for each participant according to,

$$PRST = \text{mean} \left(\frac{\text{probability targets}}{\text{probability targets} + \text{probability competitors}} \right). \quad (9)$$

In a third step, we calculated the correlations of the individual *PRSTs* between the context effects, separately for each model. Figure 7 plots these *PRSTs* for each model. As can be seen from the figure, only MDFT predicted a clear negative correlation between similarity and attraction triplets (logit model: $r = -.09$, $SD = .01$; probit model: $r = -.14$, $SD = .01$; MDFT: $r = -.53$, $SD = .05$), and both MDFT and the probit model accounted for the negative correlation between similarity and compromise triplets (logit model: $r = -.09$, $SD = .01$; probit model: $r = -.38$, $SD = .03$; MDFT: $r = -.45$, $SD = .06$). None of the models predicted a strong positive correlation between attraction and compromise triplets (logit model: $r = .12$, $SD = .01$; probit model: $r = .17$, $SD = .01$; MDFT: $r = .02$, $SD = .01$).

Next, we compared the predicted *PRST* with the observed *RST* separately for the attraction, compromise, and similarity triplets. The results indicated high correlations for MDFT (attraction: $r = .67$, $SD = .06$; compromise: $r = .81$, $SD = .12$; similarity: $r = .64$, $SD = .10$), followed by the probit (attraction: $r = .45$, $SD = .03$; compromise: $r = .48$, $SD = .06$; similarity: $r = .36$, $SD = .06$), and logit (attraction: $r = .49$, $SD = .04$; compromise: $r = .30$, $SD = .05$; similarity: $r = .35$, $SD = .06$) models.

As in Study 1, we also compared the estimated attribute weights between the models. The correlations between attribute weights of

MDFT and the probit model, between MDFT and the logit model, and between the probit and logit model ranged from .59 to .76, from .21 to .39, and from .19 to .34, respectively (see the supplemental materials for the ranges of the estimated parameters). Thus, attribute weights obtained through MDFT were more similar to the probit than to the logit model. In general, all correlations were lower than those observed in Study 1.

Discussion

In Study 2, we successfully elicited multiple context effects within single individuals. In theory, this provides a case in which MDFT should have an advantage over standard RUMs for predicting choices because MDFT can account for multiple context effects. In line with this proposition, taking model complexity into account, the decisions of most participants were best described (according to the BIC) and predicted (assessed by cross-validation) by MDFT as compared with the logit and probit models.

Correlation between context effects. Eliciting the attraction, the compromise, and the similarity effect simultaneously in a within-subject design allowed us to test for possible correlations between the three context effects. Interestingly, we observed that the attraction and compromise effects were positively correlated and that both were negatively correlated with the similarity effect. These correlations also provide a conjecture as to why it is difficult to elicit all three context effects within a single individual: If both the attraction and the compromise effect are negatively correlated with the similarity effect, then eliciting one of the first two effects also makes finding the similarity effect less likely, and vice versa.

Our results also indicate that the compromise and similarity effect were not as strong as the attraction effect. We can think of two reasons for this difference. First, the added option for compromise choice triplets might not have been placed far enough from the other options in the attribute space to be perceived as an extreme option. If so, it could explain why the target was chosen less frequently in this condition. Second, as outlined above, due to rounding, half of the similarity choice triplets became attraction choice triplets and had to be excluded from the analysis, which in turn reduced the statistical power to detect similarity effects. The only other studies investigating multiple context effects within the same person did not report the correlations between the effects, which makes it hard to assess the relative magnitude of the correlations we found (Trueblood, Brown, & Heathcote, 2013; Tsetsos et al., 2012).

Predicted correlations between context effects. The observed correlations are in accordance with the MDFT predictions that Roe and colleagues (2001) derived from a set of theoretically derived parameters. They predicted a negative correlation between the similarity effect, with both the attraction and the compromise effect, and a positive correlation between the attraction and the compromise effect, which provides further evidence for MDFT.

On the basis of the estimated models' parameters, only MDFT predicted a negative correlation between the attraction and the similarity effect. Both MDFT and the probit model predicted a negative correlation between the similarity and the compromise effect. These predictions might be misleading for the probit model. That is because even though the probit model cannot predict the single context effects (Busemeyer et al., 2007), it is nevertheless

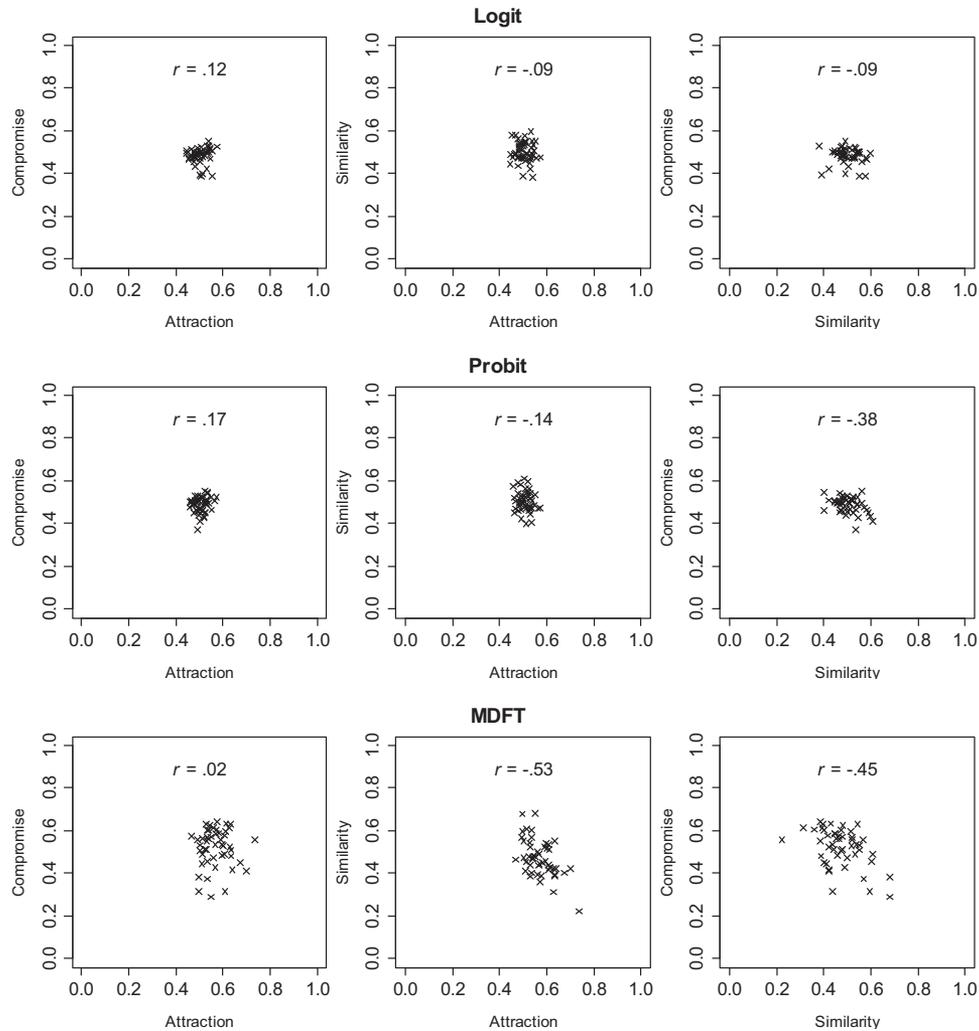


Figure 7. Comparison of the predicted relative choice shares of the target on an individual level across models and context effect. MDFT = multialternative decision field theory.

possible to find a correlation between context effects. For example, in the probit model, the *PRST* for similarity and compromise choice triplets can be lower than .50 (i.e., indicating no context effect), but can still correlate for *PRST* values below .50. Besides, Figure 7 shows rather narrowly distributed *PRST* for the probit model as compared with the more widely distributed *PRST* for MDFT, which seems to be more in line with the observed *RST* (see Figure 5). This provides further evidence for MDFT.

General Discussion

The present work followed the idea that people make choices by comparing options with each other and thereby violate the principle of independent evaluations of options. To explain these interdependent evaluations, different theories have been proposed in the past literature, including the prominent MDFT (Roe et al., 2001). The goal of our work was to show that MDFT provides an accurate empirical description of interdependent preferences and thereby outcompetes standard RUMs, such as the logit and probit

models. Even though these simple RUMs have been repeatedly criticized in the past (Busemeyer et al., 2007; Rieskamp et al., 2006), they still represent the standard approach for predicting choice behavior in economics, psychology, consumer research, and related fields (e.g., Train, 2003). At the same time, cognitive process models, such as MDFT, have been called for (e.g., Otter et al., 2008). Therefore, an empirical test of MDFT against the RUMs appeared necessary.

To make MDFT testable on the basis of empirical grounds, we used a generalized distance function that yields the similarities between options within a choice set described on different attributes weights (Berkowitsch et al., 2013). In addition to that, we also assumed that a decision is made when the preference state that develops over time converges to a constant value. Making this simplifying assumption reduces the computational effort for calculating the model's predicted choice probabilities for a large set of choice situations, as it allows for closed-form representations of the model. However, with this simplification

of the decision process, MDFT loses its ability to predict decision times.

Study 1 confirmed that MDFT can be successfully fitted to observed choice data, but in the context on hand, it did not necessarily outperform the more parsimonious logit and probit models when model complexity is taken into account. Thus, when the goal is to predict the outcome in simple preferential choice situations, such as those in Study 1, applying RUMs seems justified on pragmatic grounds, as they can be easily implemented and estimated.

In Study 2, in which we used a within-subject experimental design, we showed that MDFT outperformed RUMs in situations in which people's choices were systematically influenced by the context in which the options were presented. Presumably, this is because MDFT can account for these context effects, whereas RUMs assume that options are evaluated independently from each other. Note that nevertheless a quarter of the participants were still assigned to the logit model, suggesting that not all participants were sensitive to context effects. The experimental design of Study 2 further provided the opportunity to explore the correlation between different context effects. Toward a better understanding of these correlations, choice models need to be able to account for multiple context effects simultaneously and to describe how they emerge. Cognitive process models such as MDFT depict promising theories to predict and explain preferential choices and their underlying evaluation processes.

When creating the choice set for Study 1, we excluded choice triplets with dominant options. Therefore, we did not expect attraction effects to occur in this study. Other than that, the choice tasks were quite similar between the two studies, and thus we have no reason to believe that the cognitive processes that participants used were very different between the studies. Thus, the fact that the logit and the probit models fitted the data equally well as MDFT in Study 1 was probably due to the way we selected the choice options. However, to clearly show that the cognitive process underlying the choices in Study 1 were the same as in Study 2 requires further data that go beyond the scope of our experiment.

Advantages of Process Models

Although past research yields a considerable improvement of the simple RUMs so that some of their variants can account for systematic context effects (e.g., Kamenica, 2008; Kivetz, Netzer, & Srinivasan, 2004a, 2004b; Orhun, 2009; Roederkerk, van Heerde, & Bijmolt, 2011), these models mostly remain silent about the cognitive process underlying decision making. At the same time, a growing body of literature inside and outside psychology promotes the application of cognitive process models to better understand and predict choice behavior (e.g., Chandukala et al., 2007; Otter et al., 2008; Reutskaja et al., 2011).

In the present work, we focused on MDFT as one prominent sequential sampling model that applies to preferential choices. However, other sequential sampling models, such as the leaky competing accumulator model (LCA; Usher & McClelland, 2004) or the multi-attribute linear ballistic accumulator model (MLBA; Trueblood, Brown, & Heathcote, 2013), can also account for context effects. The LCA (Usher & McClelland, 2004) in theory also predicts a positive correlation between the attraction and compromise effect, as the same mechanism (i.e., loss aversion) is

responsible for producing the two effects (see also Tsetsos et al., 2010). Because the similarity effect is highest when loss aversion is absent, the LCA predicts negative correlations with the other two effects, which are also in line with our observations. We did not include a test of LCA or MLBA in comparison to MDFT, because as psychological process models, they are conceptually similar and they are all in contrast to standard economic models that just focus on observed outcomes.

In comparison to standard economic models, cognitive process models have additional advantages. For instance, multiple studies have shown that context effects vary over deliberation time (Dhar, Nowlis, & Sherman, 2000; Lin, Sun, Chuang, & Su, 2008; Pettibone, 2012). These findings strengthen the relevance of process models, such as MDFT and LCA. Recently, these models have also been linked to neurological processes in the brain (e.g., Forstmann et al., 2010; Gluth, Rieskamp, & Büchel, 2012, 2013). Despite these advantages, so far comparisons between these models, for example, between MDFT and LCA, have mainly relied on theoretical arguments (Pettibone, 2012). Presumably, this is the case because it has proved somewhat difficult to actually fit these models to empirical data.

To advance our understanding of the cognitive processes that govern preferential choices, it is important to compare these models on empirical grounds. Toward this goal, providing empirically testable versions of these models also allows putting them to practical use, for example, as a feasible replacement for RUMs that, despite their limitations, still represent the standard approach in many applied fields such as market research. Besides, unlike with RUMs, the application of cognitive process models is not limited to preferential choice tasks; they have also been successfully applied to perceptual (Trueblood, Brown, Heathcote, & Busemeyer, 2013), inferential (Trueblood, 2012), and risky choice tasks (Tsetsos et al., 2012).

Models like MDFT and the logit and probit models are not the end points of the complexity continuum, as more complex as well as simpler models might exist. For example, Payne, Bettmann, and Johnson (1993) provided an overview of different strategies people could follow for making preferential choices, such as a simple lexicographic heuristic that focuses only on one single attribute. However, we applied a simplified version of MDFT by assuming infinite decision time, $t \rightarrow \infty$. That means the fully specified version of MDFT (Roe et al., 2001) is more complex. The MLBA as an alternative sequential sampling model cannot easily be placed on this (one-dimensional) complexity continuum, as it additionally provides response time data. A different approach to compare the models' complexities is to apply a Bayesian approach that weights the model's possible predictions by its prior probability and additionally accounts for the functional form of the model parameters (e.g., Scheibehenne, Rieskamp, & Wagenmakers, 2013). As the attraction effect was particularly strong compared with the compromise and similarity effect, simpler models that only predict the attraction effect might have yielded comparable model fit to MDFT with less complexity.

Practical and Policy Implications

One of the reasons for the widespread use of RUMs such as the logit and probit models probably is their relative ease of imple-

mentation. Although the advantages of cognitive process models have been acknowledged, in applied contexts—such as market research—these models are rarely applied because so far, several model specifications have remained unclear (Otter et al., 2008). As our results show, cognitive process models can be readily applied to actual empirical data, indicating that they can provide a valuable alternative to RUMs to predict people's preferences. In particular, the present work specified MDFT such that its parameters could be estimated from empirical data, and choice behavior could be predicted. Because this specification came at the price of losing the ability to make predictions about decision time, future developments of MDFT should aim at further modifying the model so that all parameters can be estimated.

In an applied setting, researchers might also be interested in the subjective importance weights that people assign to specific attributes. For instance, in a consumer context, marketing companies want to infer the weights given to products' attributes. In the medical domain, physicians may want to know the importance people attach to the different aspects of a treatment, such as the treatment's success as compared with its side effects. In the educational domain, it is crucial to know how much importance teachers, parents, and pupils give to the topics taught at school. A feasible way to investigate these questions is by conducting choice studies and applying RUMs, such as the logit or probit model, to infer the importance of different aspects. However, given that decisions are systematically influenced by context effects, the estimated importance weights may be biased. In Study 1, MDFT and the RUMs highly agreed on the estimated attribute weights, providing evidence that they could be used to elicit the importance the participants gave to the different attributes. However, this agreement was much lower in Study 2. Here, the RUMs performed worse than MDFT in predicting the observed choices, so the estimated attribute weights of the RUMs in Study 2 may not necessarily reflect participants' "true" importance weights (for a discussion on estimating importance weights, see Marley, Flynn, & Louviere, 2008, and Marley & Pihlens, 2012).

Technically, RUMs can, to some extent, account for context effects by adjusting the attribute weights, which may lead to unreliable and biased results. At the same time, the interdependent evaluations of choice options and hence the occurrence of context effects is widespread: For instance, there is a large body of research on the influence of context effects on hiring decisions (Aaker, 1991; Highhouse, 1996, 1997; Slaughter, 2007; Slaughter, Bagger, & Li, 2006; Slaughter & Highhouse, 2003; Slaughter, Kausel, & Quiñones, 2011; Slaughter, Sinar, & Highhouse, 1999; Tenbrunsel & Diekmann, 2002). Likewise, context effects have been reported for perceptual (Trueblood, Brown, Heathcote, & Busemeyer, 2013), inferential (Trueblood, 2012), and risky choice tasks (Tsetsos et al., 2012). For a meta-analysis of context effects for various consumer products, see Heath and Chatterjee (1995). Together, these studies illustrate that the subjective importance weights and the predicted choice proportions by RUMs might be less reliable when context effects are likely to occur. Our results indicate that it is in these situations in which the application of cognitive models such as MDFT is probably most advantageous.

References

- Aaker, J. (1991). The negative attraction effect? A study of the attraction effect under judgment and choice. *Advances in Consumer Research*, 18, 462–469.
- Adamowicz, W., Louviere, J., & Williams, M. (1994). Combining revealed and stated preference methods for valuing environmental amenities. *Journal of Environmental Economics and Management*, 26, 271–292. doi:10.1006/jeem.1994.1017
- Attneave, F. (1950). Dimensions of similarity. *American Journal of Psychology*, 63, 516–556. doi:10.2307/1418869
- Berkowitsch, N. A. J., Scheibehenne, B., Matthäus, M., & Rieskamp, J. (2013). *Determining the similarity between multiattribute options: A generalized similarity function*. Manuscript submitted for publication.
- Bowler, S., Karp, J. A., & Donovan, T. (2010). Strategic coalition voting: Evidence from New Zealand. *Electoral Studies*, 29, 350–357. doi:10.1016/j.electstud.2010.03.001
- Browne, M. W. (2000). Cross-validation methods. *Journal of Mathematical Psychology*, 44, 108–132. doi:10.1006/jmps.1999.1279
- Busemeyer, J. R., Barkan, R., Mehta, S., & Chaturvedi, A. (2007). Context effects and models of preferential choice: Implications for consumer behavior. *Marketing Theory*, 7, 39–58. doi:10.1177/1470593107073844
- Busemeyer, J. R., & Diederich, A. (2002). Survey of decision field theory. *Mathematical Social Sciences*, 43, 345–370. doi:10.1016/S0165-4896(02)00016-1
- Busemeyer, J. R., Jessup, R. K., Johnson, J. G., & Townsend, J. T. (2006). Building bridges between neural models and complex decision making behavior. *Neural Networks*, 19, 1047–1058. doi:10.1016/j.neunet.2006.05.043
- Busemeyer, J. R., & Townsend, J. T. (1993). Decision field theory: A dynamic-cognitive approach to decision making in an uncertain environment. *Psychological Review*, 100, 432–459. doi:10.1037/0033-295X.100.3.432
- Busemeyer, J. R., & Wang, Y. (2000). Model comparisons and model selections based on generalization criterion methodology. *Journal of Mathematical Psychology*, 44, 171–189. doi:10.1006/jmps.1999.1282
- Carmon, Z., & Simonson, I. (1998). Price-quality trade-offs in choice versus matching: New insights into the prominence effect. *Journal of Consumer Psychology*, 7, 323–343. doi:10.1207/s15327663jcp0704_02
- Chandukala, S. R., Kim, J., Otter, T., Rossi, P. E., & Allenby, G. M. (2007). Choice models in marketing: Economic assumptions, challenges and trends. *Foundations and Trends in Marketing*, 2, 97–184. doi:10.1561/1700000008
- Daganzo, C. (1980). *Multinomial probit*. New York, NY: Academic Press.
- Dhar, R., Nowlis, S. M., & Sherman, S. J. (2000). Trying hard or hardly trying: An analysis of context effects in choice. *Journal of Consumer Psychology*, 9, 189–200. doi:10.1207/S15327663JCP0904_1
- Diederich, A. (2010). Mathematical psychology. In I. B. Weiner & W. E. Craighead (Eds.), *The Corsini encyclopedia of psychology* (4th ed., pp. 971–974). New York, NY: Wiley. doi:10.1002/9780470479216.corpsy0532
- Forstmann, B. U., Anwander, A., Schäfer, A., Neumann, J., Brown, S., Wagenmakers, E.-J., . . . Turner, R. (2010). Cortico-striatal connections predict control over speed and accuracy in perceptual decision making. *Proceedings of the National Academy of Sciences*, 107, 15916–15920. doi:10.1073/pnas.1004932107
- Gil, J. M., & Sánchez, M. (1997). Consumer preferences for wine attributes: A conjoint approach. *British Food Journal*, 99, 3–11. doi:10.1108/00070709710158825
- Gluth, S., Rieskamp, J., & Büchel, C. (2012). Deciding when to decide: Time-variant sequential sampling models explain the emergence of value-based decisions in the human brain. *Journal of Neuroscience*, 32, 10686–10698. doi:10.1523/JNEUROSCI.0727-12.2012

- Gluth, S., Rieskamp, J., & Büchel, C. (2013). Classic EEG motor potentials track the emergence of value-based decisions. *NeuroImage*, *79*, 394–403. doi:10.1016/j.neuroimage.2013.05.005
- Green, P. E., & Srinivasan, V. (1978). Conjoint analysis in consumer research: Issues and outlook. *Journal of Consumer Research*, *5*, 103–123. doi:10.1086/208721
- Green, P. E., & Srinivasan, V. (1990). Conjoint analysis in marketing: New developments with implications for research and practice. *Journal of Marketing*, *54*, 3–19. doi:10.2307/1251756
- Hanley, N., Wright, R. E., & Adamowicz, V. (1998). Using choice experiments to value the environment. *Environmental and Resource Economics*, *11*, 413–428. doi:10.1023/A:1008287310583
- Hausman, J. A., & Wiese, D. A. (1978). A conditional probit model for qualitative choice: Discrete decisions recognizing interdependence and heterogeneous preferences. *Econometrica*, *46*, 403–426. doi:10.2307/1913909
- Heath, T. B., & Chatterjee, S. (1995). Asymmetric decoy effects on lower-quality versus higher-quality brands: Meta analytic and experimental evidence. *Journal of Consumer Research*, *22*, 268–284. doi:10.1086/209449
- Hensher, D. A. (1994). Stated preference analysis of travel choices: The state of practice. *Transportation*, *21*, 107–133. doi:10.1007/BF01098788
- Highhouse, S. (1996). Context-dependent selection: The effects of decoy and phantom job candidates. *Organizational Behavior and Human Decision Processes*, *65*, 68–76. doi:10.1006/obhd.1996.0006
- Highhouse, S. (1997). Understanding and improving job-finalist choice: The relevance of behavioral decision research. *Human Resource Management Review*, *7*, 449–470. doi:10.1016/S1053-4822(97)90029-2
- Hotaling, J. M., Busemeyer, J. R., & Li, J. (2010). Theoretical developments in decision field theory: Comment on Tsetsos, Usher, and Chater (2010). *Psychological Review*, *117*, 1294–1298. doi:10.1037/a0020401
- Huber, J., Payne, J. W., & Puto, C. (1982). Adding asymmetrically dominated alternatives: Violations of regularity and the similarity hypothesis. *Journal of Consumer Research*, *9*, 90–98. doi:10.1086/208899
- Huber, J., & Puto, C. (1983). Market boundaries and product choice: Illustrating attraction and substitution effects. *Journal of Consumer Research*, *10*, 31–44. doi:10.1086/208943
- Johnson, J. G., & Busemeyer, J. R. (2010). Decision making under risk and uncertainty. *Wiley Interdisciplinary Reviews: Cognitive Science*, *1*, 736–749. doi:10.1002/wcs.76
- Kamenica, E. (2008). Contextual inferences in markets: On the informational content of product lines. *American Economic Review*, *98*, 2127–2149. doi:10.1257/aer.98.5.2127
- Karp, J. A. (2009). Candidate effects and spill-over in mixed systems: Evidence from New Zealand. *Electoral Studies*, *28*, 41–50. doi:10.1016/j.electstud.2008.06.012
- Kivetz, R., Netzer, O., & Srinivasan, V. (2004a). Extending compromise effect models to complex buying situations and other context effects. *Journal of Marketing Research*, *41*, 262–268. doi:10.1509/jmkr.41.3.262.35993
- Kivetz, R., Netzer, O., & Srinivasan, V. (2004b). Models for capturing the compromise effect. *Journal of Marketing Research*, *41*, 237–257. doi:10.1509/jmkr.41.3.237.35990
- Koistinen, L., Pouta, E., Heikkilä, J., Forsman-Hugg, S., Kotro, J., Mäkelä, J., & Niva, M. (2013). The impact of fat content, production methods and carbon footprint information on consumer preferences for minced meat. *Food Quality and Preference*, *29*, 126–136. doi:10.1016/j.foodqual.2013.03.007
- Kruschke, J. K. (2011a). Bayesian assessment of null values via parameter estimation and model comparison. *Perspectives on Psychological Science*, *6*, 299–312. doi:10.1177/1745691611406925
- Kruschke, J. K. (2011b). *Doing Bayesian data analysis. A tutorial with R and BUGS*. Burlington, MA: Academic Press.
- Lewandowsky, S., & Farrell, S. (2011). *Computational modeling in cognition*. Thousand Oaks, CA: Sage.
- Lin, C.-H., Sun, Y.-C., Chuang, S.-C., & Su, H.-J. (2008). Time pressure and the compromise and attraction effects in choice. *Advances in Consumer Research*, *35*, 348–352.
- Loureiro, M. L., & Umberger, W. J. (2005). Assessing consumer preferences for country-of-origin labeling. *Journal of Agricultural and Applied Economics*, *37*, 49–63.
- Loureiro, M. L., & Umberger, W. J. (2007). A choice experiment model for beef: What US consumer responses tell us about relative preferences for food safety, country-of-origin labeling and traceability. *Food Policy*, *32*, 496–514. doi:10.1016/j.foodpol.2006.11.006
- Luce, R. D. (1959). *Individual choice behavior*. Oxford, England: Wiley.
- Marley, A. A. J., Flynn, T. N., & Louviere, J. J. (2008). Probabilistic models of set-dependent and attribute-level best-worst choice. *Journal of Mathematical Psychology*, *52*, 281–296. doi:10.1016/j.jmp.2008.02.002
- Marley, A. A. J., & Pihlens, D. (2012). Models of best-worst choice and ranking among multiattribute options (profiles). *Journal of Mathematical Psychology*, *56*, 24–34. doi:10.1016/j.jmp.2011.09.001
- McClelland, J. L., & Rumelhart, D. E. (1981). An interactive activation model of context effects in letter perception: Part 1. An account of basic findings. *Psychological Review*, *88*, 375–407. doi:10.1037/0033-295X.88.5.375
- McFadden, D. (1973). Conditional logit analysis of qualitative choice behavior. In P. Zarembka (Ed.), *Frontiers in econometrics* (pp. 105–142). New York, NY: Academic Press.
- McFadden, D. (2001). Economic choices. *American Economic Review*, *91*, 351–378. doi:10.1257/aer.91.3.351
- Menard, S. (2004). Six approaches to calculating standardized logistic regression coefficients. *American Statistician*, *58*, 218–223. doi:10.1198/000313004X946
- Mosteller, F., & Noguee, P. (1951). An experimental measurement of utility. *Journal of Political Economy*, *59*, 371–404. doi:10.1086/257106
- Nownes, A. J. (1992). Primaries, general elections, and voter turnout: A multinomial logit model of the decision to vote. *American Politics Research*, *20*, 205–226. doi:10.1177/1532673X9202000204
- Orhun, A. Y. (2009). Optimal product line design when consumers exhibit choice-set dependent preferences. *Marketing Science*, *28*, 868–886. doi:10.1287/mksc.1080.0449
- Otter, T., Johnson, J., Rieskamp, J., Allenby, G. M., Brazell, J. D., Diederich, A., . . . Townsend, J. (2008). Sequential sampling models of choice: Some recent advances. *Marketing Letters*, *19*, 255–267. doi:10.1007/s11002-008-9039-0
- Payne, J. W., Bettman, J. R., & Johnson, E. (1993). *The adaptive decision maker*. Cambridge, England: Cambridge University Press. doi:10.1017/CBO9781139173933
- Pettibone, J. C. (2012). Testing the effect of time pressure on asymmetric dominance and compromise decoys in choice. *Judgment and Decision Making*, *7*, 513–523.
- Phillips, K. A., Maddala, T., & Johnson, F. R. (2002). Measuring preferences for health care interventions using conjoint analysis: An application to HIV testing. *Health Services Researches*, *37*, 1681–1705. doi:10.1111/1475-6773.01115
- Raftery, A. E. (1995). Bayesian model selection in social research. *Sociological Methodology*, *25*, 111–163. doi:10.2307/271063
- Ratcliff, R. (1978). A theory of memory retrieval. *Psychological Review*, *85*, 59–108. doi:10.1037/0033-295X.85.2.59
- Reutskaja, E., Nagel, R., Camerer, C. F., & Rangel, A. (2011). Search dynamics in consumer choice under time pressure: An eye-tracking study. *American Economic Review*, *101*, 900–926. doi:10.1257/aer.101.2.900
- Rieskamp, J. (2008). The probabilistic nature of preferential choice. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *34*, 1446–1456. doi:10.1037/a0013646

- Rieskamp, J., Busemeyer, J. R., & Mellers, B. A. (2006). Extending the bounds of rationality: Evidence and theories of preferential choice. *Journal of Economic Literature*, *44*, 631–661. doi:10.1257/jel.44.3.631
- Roberts, D. C., Boyer, T. A., & Lusk, J. L. (2008). Preferences for environmental quality under uncertainty. *Ecological Economics*, *66*, 584–593. doi:10.1016/j.ecolecon.2008.05.010
- Roe, R. M., Busemeyer, J. R., & Townsend, J. T. (2001). Multialternative decision field theory: A dynamic connectionist model of decision making. *Psychological Review*, *108*, 370–392. doi:10.1037/0033-295X.108.2.370
- Rooderkerk, R. P., van Heerde, H. J., & Bijmolt, T. H. A. (2011). Incorporating context effects into a choice model. *Journal of Marketing Research*, *48*, 767–780. doi:10.1509/jmkr.48.4.767
- Ryan, M., & Farrar, S. (2000). Using conjoint analysis to elicit preferences for health care. *BMJ: British Medical Journal*, *320*, 1530–1533. doi:10.1136/bmj.320.7248.1530
- Scheibehenne, B., Rieskamp, J., & Gonzalez-Vallejo, C. (2009). Cognitive models of choice: Comparing decision field theory to the proportional difference model. *Cognitive Science*, *33*, 911–939. doi:10.1111/j.1551-6709.2009.01034.x
- Scheibehenne, B., Rieskamp, J., & Wagenmakers, E.-J. (2013). Testing adaptive toolbox models: A Bayesian hierarchical approach. *Psychological Review*, *120*, 39–64. doi:10.1037/a0030777
- Simonson, I., & Tversky, A. (1992). Choice in context: Tradeoff contrast and extremeness aversion. *Journal of Marketing Research*, *29*, 281–295. doi:10.2307/3172740
- Slaughter, J. E. (2007). Effects of two selection batteries on decoy effects in job-finalist choice. *Journal of Applied Social Psychology*, *37*, 76–90. doi:10.1111/j.0021-9029.2007.00148.x
- Slaughter, J. E., Bagger, J., & Li, A. (2006). Context effects on group-based employee selection decisions. *Organizational Behavior and Human Decision Processes*, *100*, 47–59. doi:10.1016/j.obhdp.2006.01.003
- Slaughter, J. E., & Highhouse, S. (2003). Does matching up features mess up job choice? Boundary conditions on attribute-salience effects. *Journal of Behavioral Decision Making*, *16*, 1–15. doi:10.1002/bdm.428
- Slaughter, J. E., Kausel, E. E., & Quiñones, M. A. (2011). The decoy effect as a covert influence tactic. *Journal of Behavioral Decision Making*, *24*, 249–266. doi:10.1002/bdm.687
- Slaughter, J. E., Sinar, E. F., & Highhouse, S. (1999). Decoy effects and attribute-level inferences. *Journal of Applied Psychology*, *84*, 823–828. doi:10.1037/0021-9010.84.5.823
- Slovic, P., & Tversky, A. (1974). Who accepts Savage's axiom? *Behavioral Science*, *19*, 368–373. doi:10.1002/bs.3830190603
- Soltani, A., De Martino, B., & Camerer, C. (2012). A range-normalization model of context-dependent choice: A new model and evidence. *PLOS Computational Biology*, *8*, e1002607. doi:10.1371/journal.pcbi.1002607
- Stone, M. (1974). Cross-validated choice and assessment of statistical predictions. *Journal of the Royal Statistical Society*, *36B*, 111–147.
- Tenbrunsel, A. E., & Diekmann, K. A. (2002). Job decision inconsistencies involving social comparison information: The role of dominating alternatives. *Journal of Applied Psychology*, *87*, 1149–1158. doi:10.1037/0021-9010.87.6.1149
- Thurstone, L. L. (1927). A law of comparative judgment. *Psychological Review*, *101*, 266–270. doi:10.1037/0033-295X.101.2.266
- Townsend, J. T., & Ashby, F. G. (1983). *Stochastic modeling of elementary psychological processes*. Cambridge, England: Cambridge University Press.
- Train, K. (1978). A validation test of a disaggregate mode choice model. *Transportation Research*, *12*, 167–174. doi:10.1016/0041-1647(78)90120-X
- Train, K. E. (2003). *Discrete choice methods with simulation*. Cambridge, England: Cambridge University Press. doi:10.1017/CBO9780511753930
- Trueblood, J. S. (2012). Multialternative context effects obtained using an inference task. *Psychonomic Bulletin & Review*, *19*, 962–968. doi:10.3758/s13423-012-0288-9
- Trueblood, J. S., Brown, S. D., & Heathcote, A. (2013). The multi-attribute linear ballistic accumulator model of decision-making. In M. Knauff, M. Pauen, N. Sebanz, & I. Wachsmuth (Eds.), *Proceeding of the 35th Annual Conference of the Cognitive Science Society* (pp. 3581–3586). Austin, TX: Cognitive Science Society.
- Trueblood, J. S., Brown, S. D., Heathcote, A., & Busemeyer, J. R. (2013). Not just for consumers: Context effects are fundamental to decision making. *Psychological Science*, *24*, 901–908. doi:10.1177/0956797612464241
- Tsetsos, K., Chater, N., & Usher, M. (2012). Salience driven value integration explains decision biases and preference reversals. *Proceedings of the National Academy of Sciences*, *109*, 9659–9664. doi:10.1073/pnas.1119569109
- Tsetsos, K., Usher, M., & Chater, N. (2010). Preference reversal in multiattribute choice. *Psychological Review*, *117*, 1275–1291. doi:10.1037/a0020580
- Tversky, A. (1972a). Choice by elimination. *Journal of Mathematical Psychology*, *9*, 341–367. doi:10.1016/0022-2496(72)90011-9
- Tversky, A. (1972b). Elimination by aspects: A theory of choice. *Psychological Review*, *79*, 281–299. doi:10.1037/h0032955
- Tversky, A., Sattah, S., & Slovic, P. (1988). Contingent weighting in judgment and choice. *Psychological Review*, *95*, 371–384. doi:10.1037/0033-295X.95.3.371
- Tversky, A., & Simonson, I. (1993). Context-dependent preferences. *Management Science*, *39*, 1179–1189. doi:10.1287/mnsc.39.10.1179
- Usher, M., & McClelland, J. L. (2004). Loss aversion and inhibition in dynamical models of multialternative choice. *Psychological Review*, *111*, 757–769. doi:10.1037/0033-295X.111.3.757
- Vickers, D. (1970). Evidence for an accumulator of psychophysical discrimination. *Ergonomics*, *13*, 37–58. doi:10.1080/00140137008931117
- von Neumann, J., & Morgenstern, O. (1947). *Theory of games and economic behavior* (2nd ed.). Princeton, NJ: Princeton University Press.
- Wardman, M. (1988). A comparison of revealed preference and stated preference models of travel behavior. *Journal of Transport Economics and Policy*, *22*, 71–91.
- Wedell, D. H. (1991). Distinguishing among models of contextually induced preference reversals. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *17*, 767–778. doi:10.1037/0278-7393.17.4.767

(Appendices follow)

Appendix A

Simplifying the Stopping Rule

In its original form, MDFT provides two explanations of how a decision is reached: Either an internal stopping rule leads to a decision or an external stopping rule limits the decision process. The *internal stopping rule* assumes that a decision is made as soon as the accumulated preference state for one option reaches a threshold θ . However, if a too high θ is set, the threshold might never be reached, because the preference states might have converged to a stable value before reaching θ . On the other hand, setting θ too low can lead to preference states that have not yet converged, and further evidence would potentially yield different choice probabilities. In theory, one could keep track of the change in choice probabilities to stop the estimation process as soon as convergence is reached. This is computationally unsatisfying, though, because obtaining stable choice probabilities is cumbersome. As an alternative, one can assume an *external stopping rule* with unconstrained deliberation time (i.e., a very high t). However, the associated time-intensive simulations and the required computational effort to fit MDFT are still unsatisfying (Trueblood, Brown, & Heathcote, 2013), as the preference state \mathbf{P} needs to be iterated until t is reached for every set of tested parameters. A better way to think of decisions with no time constraints is to set $t \rightarrow \infty$, which according to Roe et al. (2001) reduces the calculation of the mean preference state over time $\xi(t)$ to:

$$\xi^{(\infty)} = (\mathbf{I} - \mathbf{S})^{-1}\mu. \tag{A1}$$

Thus, instead of iterating for a very long time to calculate $\xi(t)$, we can directly calculate $\xi^{(\infty)}$. Deriving choice probabilities for $t \rightarrow \infty$ further requires the variance–covariance matrix of the preference state. Busemeyer, Jessup, Johnson, and Townsend (2006) suggested a formula for $\Omega^{(\infty)}$; however, this solution is limited to cases where the variance–covariance matrix of \mathbf{V} is a diagonal matrix, that is, $\Phi = \varphi^2 \cdot \mathbf{I}$. In the next section, we develop a general formula to directly calculate $\Omega^{(\infty)}$ for k options. This avoids time-consuming calculations of the variance–covariance matrix for each time point, and it leads to stable choice predictions. The variance–covariance matrix at time t is calculated as:

$$\Omega_t = \sum_{j=0}^{t-1} \mathbf{S}^j \Phi \cdot (\mathbf{S}^j)', \tag{A2}$$

where Φ is the $k \times k$ variance–covariance matrix of \mathbf{V} (see Roe et al., 2001, Appendix B). From complete induction follows,

$$\Omega_{t+1} = \mathbf{S} \cdot \Omega_t \cdot \mathbf{S}' + \Phi. \tag{A3}$$

By combining Equations A2 and A3, we can calculate the change in the variance–covariance matrix after one iteration (i.e., $t + 1$) by

$$\Omega_{t+1} - \Omega_t = \mathbf{S} \cdot \Omega_t \cdot \mathbf{S}' + \Phi - \Omega_t. \tag{A4}$$

For the feedback matrix \mathbf{S} with eigenvalues smaller than 1, the sequence of the matrices \mathbf{S}^t for $t \rightarrow \infty$ converges to a zero matrix. We can therefore neglect the term \mathbf{S}^t after a certain (high enough) number of iterations, say, t_0 . That is,

$$\Omega_{t_0+j} = \Omega_{t_0} \quad \forall j \geq 0, \tag{A5}$$

from which follows,

$$0 = \Omega_{t_0+1} - \Omega_{t_0} = \mathbf{S} \cdot \Omega_{t_0} \cdot \mathbf{S}' + \Phi - \Omega_{t_0}. \tag{A6}$$

This means that the covariance matrix of the preference state has converged, and $\Omega_{t_0} = \Omega^{(\infty)}$. We can reorganize Equation A6 as,

$$\Omega^{(\infty)} - \mathbf{S} \cdot \Omega^{(\infty)} \cdot \mathbf{S}' = \Phi. \tag{A7}$$

Now we solve Equation A7 for $\Omega^{(\infty)}$ to obtain a system of linear equations. This is achieved by the following steps. We first explicitly calculate $\mathbf{S} \cdot \Omega^{(\infty)} \cdot \mathbf{S}'$. We next transform the $k \times k$ matrices $\mathbf{S} \cdot \Omega^{(\infty)} \cdot \mathbf{S}'$ and $\Omega^{(\infty)}$ into the $k^2 \times 1$ vectors $\overline{\mathbf{S} \cdot \Omega^{(\infty)} \cdot \mathbf{S}'}$ and $\overline{\Omega^{(\infty)}}$, respectively, so we can search the $k^2 \times k^2$ matrix \mathbf{Z} , which multiplied by $\overline{\Omega^{(\infty)}}$ is equivalent to $\overline{\mathbf{S} \cdot \Omega^{(\infty)} \cdot \mathbf{S}'}$, so that

$$\overline{\mathbf{S} \cdot \Omega^{(\infty)} \cdot \mathbf{S}'} = \mathbf{Z} \cdot \overline{\Omega^{(\infty)}}. \tag{A8}$$

Restructuring Equation A7 according to Equation A8 leads to

$$\overline{\Omega^{(\infty)}} - \mathbf{Z} \cdot \overline{\Omega^{(\infty)}} = \overline{\Phi}, \tag{A9}$$

where the $k^2 \times 1$ vector $\overline{\Phi}$ is the $k \times k$ transformed matrix Φ . Equation A9 can be solved for $\overline{\Omega^{(\infty)}}$:

$$\overline{\Omega^{(\infty)}} = (\mathbf{I} - \mathbf{Z})^{-1} \cdot \overline{\Phi}. \tag{A10}$$

Finally, we retransform the $k^2 \times 1$ vector $\overline{\Omega^{(\infty)}}$ back into the $k \times k$ matrix $\Omega^{(\infty)}$. Now we have an analytical solution for $\xi^{(\infty)}$ and for $\Omega^{(\infty)}$, so that the choice probabilities for $t \rightarrow \infty$ can be directly derived.

(Appendices continue)

Appendix B

Mathematical Formalization of the Generalized Distance Function

In the following, we provide the mathematical formalization of the generalized distance function (described in detail in Berkowitz et al., 2013). This generalized distance function is meant to describe the distance between options in the multiattribute space, distinguishes the preferential relationship between options, and accounts for individual differences by incorporating the subjective weights that individuals give to different attributes in the distance function.

We define an importance weight vector \mathbf{W} , which contains the individual weights of the n attributes and restricts the weights to sum to 1. Further, each indifference vector $\{\mathbf{iv}_j\}_{j=1}^{n-1}$ is an n -dimensional vector and can be calculated as,

$$\mathbf{iv}_j = \begin{bmatrix} -\frac{w_{j+1}}{w_1} \\ 0 \\ \vdots \\ 0 \\ \frac{w_1}{w_1} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{w_{j+1}}{w_1} \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \forall j = 1, \dots, n - 1, \tag{B1}$$

where 1 is at the $(j + 1)^{\text{th}}$ position.

Because we want the n -dimensional dominance vector \mathbf{dv} to be orthogonal to all $n - 1$ indifference vectors, it fulfills:

$$\mathbf{iv}_j \cdot \mathbf{dv} = 0 \quad \forall j = 1, \dots, n - 1, \tag{B2}$$

which leads to the generalized form:

$$\mathbf{dv} = \begin{bmatrix} \frac{w_1}{w_1} \\ \frac{w_1}{w_1} \\ \frac{w_2}{w_1} \\ \vdots \\ \frac{w_j}{w_1} \\ \vdots \\ \frac{w_n}{w_1} \\ \frac{w_1}{w_1} \end{bmatrix}. \tag{B3}$$

Now we can build the $n \times n$ basis \mathbf{B}^* , containing the $n - 1$ indifference vectors \mathbf{iv}_1 to \mathbf{iv}_{n-1} and the dominance vector \mathbf{dv} ,

$$\mathbf{B}^* = [\mathbf{iv}_1, \dots, \mathbf{iv}_j, \dots, \mathbf{iv}_{n-1}, \mathbf{dv}]. \tag{B4}$$

To standardize the lengths of the indifference vectors and the dominance vector to 1, each vector is divided by its Euclidean lengths l_{iv} and l_{dv} , where $\{l_{iv,j}\}_{j=1}^{n-1}$:

$$l_{iv_j} = \|\mathbf{iv}_j\|_2 \quad \forall j = 1, \dots, n - 1 \tag{B5}$$

and

$$l_{dv} = \|\mathbf{dv}\|_2. \tag{B6}$$

Thus, we obtain new basis \mathbf{B} , which is,

$$\mathbf{B} = \left[\frac{\mathbf{iv}_1}{l_{iv_1}}, \dots, \frac{\mathbf{iv}_j}{l_{iv_j}}, \dots, \frac{\mathbf{iv}_{n-1}}{l_{iv_{n-1}}}, \frac{\mathbf{dv}}{l_{dv}} \right], \tag{B7}$$

where the $n \times n$ matrix \mathbf{B} contains the standardized indifference vectors and the standardized dominance vector.

Next, we define the standard distance vector \mathbf{dist}_{stand} as the trajectory path between two points, expressed in standard unit vectors. To transform \mathbf{dist}_{stand} into the new distance vector \mathbf{dist}_{trans} , which expresses the trajectory path by the previously introduced indifference vectors and by the dominance vector, we make a change of basis:

$$\mathbf{dist}_{trans} = \mathbf{B}^{-1} \cdot \mathbf{dist}_{stand}. \tag{B8}$$

The first $n - 1$ entries of \mathbf{dist}_{trans} express the distance in units of each \mathbf{iv}_j , whereas the last entry of \mathbf{dist}_{trans} expresses the distance in units of \mathbf{dv} . Now we need to calculate the Euclidean length \mathbf{D}^2 of \mathbf{dist}_{trans} and multiply the distance in the dominance direction by a parameter $w\mathbf{d} > 1$. This assures that the distance in the dominance direction is weighted more strongly than the distance in the indifference directions. This is computed as follows:

$$\mathbf{D}^2 = \mathbf{dist}_{trans} \cdot \mathbf{A} \cdot \mathbf{dist}_{trans}, \tag{B9}$$

where \mathbf{A} is a $n \times n$ diagonal matrix and is constructed in the following way:

$$\mathbf{A}_{j,j} = \begin{cases} 1, & \text{if } j = 1, \dots, n - 1 \\ w\mathbf{d}, & \text{if } j = n \end{cases}. \tag{B10}$$

This assures that only the difference in the dominance direction—the last column of \mathbf{dist}_{trans} —is weighted by $w\mathbf{d}$. By setting \mathbf{A} to the identity matrix (i.e., $w\mathbf{d} = 1$), one obtains the standard Euclidean norm.

(Appendices continue)

Appendix C

Constraints on Model Parameters

For the logit model, we estimated an importance weight for each of the n attributes, denoted w_i . Here, attribute weights were allowed to vary between 0 and ∞ . For the probit model, we estimated $n - 1$ attribute weights, and the variance ν of the normal distributed error component ϵ in the diagonal of the variance-covariance matrix. Attribute weights were restricted to sum to 1, and the ν was allowed to vary between 0 and 1,000. Finally, for MDFT, we estimated the $n - 1$ attribute weights, restricted to sum to 1, the variance ν of the normal distributed error component ϵ , restricted to values between 0 and 1,000, the sensitivity parameter ϕ_1 , restricted to vary between 0.01 and 1,000, and the decay parameter ϕ_2 , restricted to values between 0 and 1. In Study 1, we fixed the weight parameter wd , which weights distances in the dominance direction relative to distances in the indifference direction, to a value of 12, following [Hotelling et al. \(2010\)](#). In Study 2, wd was estimated from the data. Here, the value range was restricted to

values between 1 and 50 (see the supplemental materials for the estimated ranges of all parameters). Prior to parameter estimation, we rescaled the range of the attributes to values between 0 and 1 and recoded all attributes such that higher numbers indicate higher values.

To compute the likelihoods of the models, we applied the multivariate normal distribution for MDFT and the probit model and the multivariate logistic distribution for the logit model and minimized the respective summed log likelihoods for each participant. To search the best fitting parameters, we used maximum likelihood methods implemented in the *R* functions “nlminb” (package: stats) and “psoptim” (package: pso).

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